

Thermodynamics and Statistical Physics

Part I – Thermodynamics

Exam T2

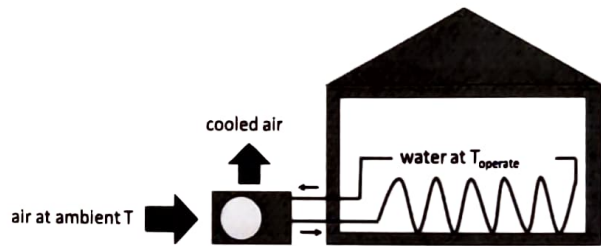
Wednesday, November 11 2021, 8:30-11:30, Aletta Jacobshal

The total number of points that can be reached in this exam is 90.

Final grade = (points/10) + 1.

1) Heat pump (30p)

An important part of our personal CO₂ emission footprint is due to heating of residential houses that is based on fossil fuels. CO₂-reduction can be accomplished by



replacing conventional heating systems with heat pumps. The figure shows the example of an air-water heatpump, where a thermodynamic cycle extracts heat from ambient air and deposits heat into the water-filled floor heating system inside the house using electrical energy. Assume that the heat pump is a reversible thermodynamical cycle based on the Carnot process. The air outside and the water of the floor-heating system are the two reservoirs.

- The efficiency of a heat pump is called “coefficient of performance” (COP). Give the general definition for the COP of a heat pump. **(6 pt)**
- Calculate COPs for the heatpump with a water temperature T_{water} of 40°C (low temperature floor heating) and for outside air temperatures of $T_{\text{air}} = -10^\circ\text{C}$, 0°C and $+10^\circ\text{C}$. Repeat the calculations for $T_{\text{water}} = 70^\circ\text{C}$ (conventional high temperature radiators). Make a table. (hint: you need the efficiency of the Carnot cycle for this). Discuss the table in terms of realistic suitability of heat pumps for reduction of CO₂ emission. **(8 pt)**
- Draw a P-V diagram of the thermodynamic cycle and indicate in which steps heat and work are exchanged with the environment. **(8 pt)**
- Calculate the entropy change inside and outside the house that is occurring for 1 kWh of electrical energy going into the heat pump as work (assume $T_{\text{air}} = 0^\circ\text{C}$ and $T_{\text{air}} = 40^\circ\text{C}$). **(8 pt)**

2) Heat transport (30p)

Assume Your apartment had a very old window of area $A = 2 \text{ m}^2$ with thermal conductance $\frac{\kappa}{L} = 6 \text{ Wm}^{-2}\text{K}^{-1}$. To save energy, this window is replaced by a state of the art double-pane window with $\frac{\kappa}{L} = 1 \text{ Wm}^{-2}\text{K}^{-1}$.

- a) Use the thermal diffusion equation in one dimension

$$\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial x^2}$$

To determine a functional form of the temperature profile through the window $T(x)$, with the temperatures T_{outside} and T_{inside} both constant. **(8 pt)**

- b) Give a functional form for the heat flux through the window using $T(x)$. **(8 pt)**
c) How much energy do you save per winter day (24 hours), assuming a constant temperature outside of -5°C and inside of 20°C ? **(8 pt)**
d) In standard double-pane windows, the volume between the two glass panes is filled with air (mostly $^{14}\text{N}_2$). The thermal conductance can be lowered even more, by filling this volume with ^{40}Ar . Why is the thermal conductance lower for ^{40}Ar ? Give two reasons. **(6 pt)**
- 3) **Spontaneous? (8 pt)**
Give two examples for processes occurring spontaneously and two examples for processes occurring non-spontaneously. (Do not simply reverse a given process to have spontaneous and non-spontaneous, I want 4 different processes!)

4) **Entropy (7 pt)**

Apply your knowledge of entropy to explain why heat flows from hot to cold.

5) **Isothermal Expansion (15pt)**

Consider a piston filled with 1 l of Ar at $T=273.15\text{ K}$ and $p=1 \times 10^5\text{ Pa}$. The gas can be considered an ideal gas.

- a) The gas in the piston expands isothermally against a constant external pressure of $p=2 \times 10^5\text{ Pa}$. Determine the change in internal energy, the work done by the gas and the heat flow. **(5pt)**
b) Repeat the calculation for a reversible isothermal expansion. Why is there a difference in work between a) and b)? **(10pt)**

Constants:

Avogadro's number: $N_A=6.02 \cdot 10^{23}\text{ mol}^{-1}$

Boltzmann constant: $k_B=1.381 \cdot 10^{-23}\text{ J/K}$

Gas constant: $R=8.31\text{ JK}^{-1}\text{mol}^{-1}$

Atomic mass unit (u): $m_u=1.67 \cdot 10^{-27}\text{ kg}$

Electronvolt: $1\text{ eV}=1.6 \cdot 10^{-19}\text{ J}$